COMPOSITIONAL SCHEDULABILITY ANALYSIS OF WORKFLOW USING TIMING CONSTRAINT PETRI NETS

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ABSTRACT
The schedulability analysis is important in workflow modeling. This paper presents a new approach to the schedulability analysis of individual transition or transition sequences in Time constraint Petri nets (TCPNs). The reachability of markings can also be checked based on the schedulability analysis. If a specific transition sequence is schedulable, the corresponding task sequence can complete its execution successfully; otherwise, nonschedulable transitions should be pinpointed to help adjust timing constraints. A technique for compositional timing analysis is also proposed to deal with complex transition sequences, which improves efficiency of analysis. We have also constructed an automated tool: TCPN-PIPE2 to facilitate the schedulability analysis of workflow.

KEY WORDS
Workflow modeling, Timing constraints Petri nets, Reachability, Compositional schedulability, TCPN-PIPE2

1 Introduction

Managing time is important in designing and managing workflow, a workflow model requires to deal with time issues. The existing models for managing time are mainly based on workflow graphs and Petri nets. Based on Workflow graphs, Eder *et al.* [3], Marjanovic [4] and Zhuge Hai *et al.* [5] proposed their own workflow models dealing with time issues.

Petri nets have gained many applications in the field of workflow modeling [6, 7], since basic Petri nets lack the temporal description, several extended models of Petri nets considering time were proposed, such as Timed Petri nets (Timed PNs) [8] and Time Petri nets (Time PNs) [2, 9].

Our work studies schedulability analysis of workflow modeled in Timing Constraint Petri nets (TCPNs) which were proposed by Tsai *et al.* [1]. As a visual model, TCPNs are very expressive to depict time constraints; Furthermore, TCPNs follow the weak firing rule [1], which is more appropriate for modeling workflow than strong firing mode that Time (Timed) PNs use. Though TCPNs have some advantages for modeling workflow, the formulas of EFBT and LFET proposed in [1] are inconsistent with the meanings of timing constraints; moreover, the approach to schedulability analysis introduced by Tsai *et al.* is insufficient to determine schedulability of individual transition. These two shortcomings, in some respects, limit the usefulness of TCPNs.

According to new formulas of EFBT and LFET different from those presented in [1], our main results include: 1) a new approach to schedulability analysis of individual transition and transition sequences, 2) a compositional strategy for complex transition sequences, 3) an automated tool for building, simulating and analyzing TCPN models. This paper is organized as follows: Section 2 gives a brief introduction to TCPNs. Section 3 presents the schedulability analysis of individual transition and transition sequences. Section 4 describes the compositional schedulability analysis. Section 5 introduces the automated tool TCPN-PIPE2 and its application. Conclusions are drawn in Section 6.

2 Timing Constraints Petri Nets and Workflow

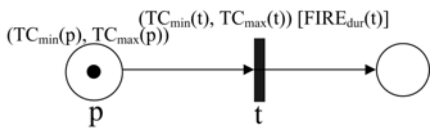
2.1 Timing Constraint Petri Nets

Definition 1. [1] A timing constraint Petri net is a 6-tuple (P, T, F, C, D, M_0) where:

- P is a finite set of places.
- T is a finite set of transitions.
- F is a set of arcs which connect places and transitions.
- C is a set of integer pairs, $(\text{TC}_{\text{min}}(pt_j), \text{TC}_{\text{max}}(pt_j))$, where pt_j is either a place or a transition.
- D is a set of integer pairs, $[\text{FIRE}_{\text{dur}}(t_j)]$, where t_j is a transition.
- M_0 is the initial marking function.

Without considering time constraints, P, T, F, and M_0 together define a Petri net, which could be denoted as the underlying net UN= $\{P, T, F, M_0\}$. Given a marking M and a place p, $M(p)$ denote the number of tokens contained in p under the marking M.

In TCPN, $\text{TC}_{\text{min}}(p)$/$\text{TC}_{\text{max}}(p)$ are the minimum/maximum elapsed time intervals between the



(TC$_{min}$(p), TC$_{max}$(p))
(TC$_{min}$(t), TC$_{max}$(t)) [FIRE$_{dur}$(t)]
p
t

Figure 1. Time constraints of TCPN

token arrival time of p and the beginning/ending firing time of p's output transitions. A transition t with a time pair, $(\text{TC}_{\text{min}}(t), \text{TC}_{\text{max}}(t))$, is said to be enabled under marking M if $(p \in \bullet t)M(p) \geq 1$. A transition t, which is enabled at time T_0, is said to be firable during the time interval $[T_0 + \text{TC}_{\text{min}}(t), T_0 + \text{TC}_{\text{max}}(t)]$. Here what we should pay much attention on is that there is no guarantee that a firable transition t can complete the firing successfully in that the firing should take a period of time $[\text{FIRE}_{\text{dur}}(t)]$. For any place or transition which has non-explicit timing constraints, the default value of a time pair is $(0, \infty)$ and the default of transition duration is 0. Fig. 1 shows time constraints in TCPN.

In comparison of other existing time-related Petri nets such as Time or Timed PNs, TCPNs have three main advantages for modeling workflow: 1) TCPNs include more expressive time constraints, in TCPN, given a workflow schema, a workflow designer can assign execution duration $\text{FIRE}_{\text{dur}}(t)$ as well as relative executable interval $(\text{TC}_{\text{min}}(t)$,$\text{TC}_{\text{max}}(t))$ for each task or transition. Whereas Timed PNs lack time pairs to restrict the execution of a transition, Time PNs can merely depict the transition which is instantaneous. In some cases, the resources or conditions modeled by places should also be limited under a time interval, the place time pair $(\text{TC}_{\text{min}}(p), \text{TC}_{\text{max}}(p))$ in TCPN can fulfill this requirement. 2) Different from Timed and Time PNs, TCPNs follow the weak firing mode. According to [1], the strong firing mode may cause two conflict transitions to fire simultaneously, which contradicts the definition of conflict structures; it also might be prone to cause dead transitions. 3) TCPNs are most suitable for systems with conflict structures.

3 Schedulabililty Analysis Using TCPN

3.1 Basic Concepts

Here, we defined some other time constraints, $\text{TOKEN}_{\text{arr}}(p)$ is an absolute time representing the time when the token arrives at place p; $\text{FIRE}_{\text{begin}}(t)$/$\text{FIRE}_{\text{end}}(t)$ is the time when transition t begins/ends the firing; EFBT(t)/LFET(t) denote the earliest firing beginning time and the latest firing ending time of transition t after t is enabled; IT(p)/OT(p) is a set of input/output transitions of p; IP(t)/OP(t) is a set of input/output places of t.

Definition 2. A transition t_j is said to be *strongly firable* if each of the input places of t_j currently has at least one token at the same time with the consideration of the actual arrival time of tokens in each input place, That is, t_j is strongly firable if and only if:

$$\begin{aligned}\text{EFBT}(t) = \text{Max}\{&\text{TOKEN}_{\text{arr}}(p_j) + \text{TC}_{\text{min}}(p_j), \\ &\text{Max}\{\text{TOKEN}_{\text{arr}}(p_j) : p_j \in \text{IP}(t_j)\} + \text{TC}_{\text{min}}(t_j)\}\end{aligned} \tag{3.1}$$

$$\begin{aligned}\text{LFET}(t) = \text{Min}\{&\text{TOKEN}_{\text{arr}}(p_j) + \text{TC}_{\text{max}}(p_j), \\ &\text{Max}\{\text{TOKEN}_{\text{arr}}(p_j) : p_j \in \text{IP}(t_j)\} + \text{TC}_{\text{max}}(t_j)\}\end{aligned} \tag{3.2}$$

and

$$\text{LFET}(t_j) - \text{EFBT}(t_j) \geq 0.$$

Definition 3. A transition t_j is *strongly schedulable* if t_j is strongly firable and can complete firing successfully with considering the arrival time of tokens in each input place. That is, t_j is strongly schedulable if and only if:

$$\text{LFET}(t_j) - \text{EFBT}(t_j) \geq \text{FIRE}_{\text{dur}}(t_j).$$

The formulas (3.1) and (3.2) were proposed by Dianxiang Xu *et al.* [2], they described that EFBT(t_j) and LFET(t_j) in [1] are inconsistent with the meanings of time constraints in TCPN.

3.2 Schedulability Analysis of A Transition

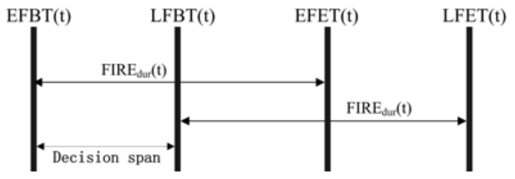


Figure 2. Relation of timing constraints

Theorem 1. If a transition t is strongly schedulable and needs to complete the firing successfully, then EFBT$(t) \leq \text{FIRE}_{\text{begin}}(t) \leq$ LFBT(t). where LFBT(t) represents the latest beginning firing time of t, LFBT(t) = LFET$(t) - \text{FIRE}_{\text{dur}}(t)$.

Proof. If a transition t is strongly schedulable, obviously, LFET(t) − EFBT$(t) \geq \text{FIRE}_{\text{dur}}(t)$, and if $\text{FIRE}_{\text{begin}}(t) >$ LFBT(t), then $\text{FIRE}_{\text{end}}(t) = \text{FIRE}_{\text{begin}}(t) + \text{FIRE}_{\text{dur}}(t) >$ LFET(t), which means that t can not complete the firing, thus we can directly draw the conclusion. □

Based on $\text{FIRE}_{\text{begin}}(t)$, $\text{FIRE}_{\text{end}}(t) \in [\text{EFET}(t), \text{LFET}(t)]$, where EFET$(t)$ represents the earliest ending firing time of t, and EFET(t)=EFBT(t) + $\text{FIRE}_{\text{dur}}(t)$. The time interval $[\text{EFBT}(t), \text{LFBT}(t)]$ can be considered as the decision span of a transition at run-time, the relation of these time constraints can be illustrated via Fig. 2.

Theorem 2. For $\forall p_j \in \text{OP}(t_i)$, if t_i completes the firing successfully, then $\text{TOKEN}_{\text{arr}}(p_j) = \text{FIRE}_{\text{end}}(t_i) \in [\text{EFET}(t_i), \text{LFET}(t_i)]$.

Proof. Because TOKEN$_{\text{arr}}(p_j)$ = FIRE$_{\text{end}}(t_i)$, if t_i completes its firing successfully, we can directly get this Theorem. $\square$

Definition 4. For $\forall p_j \in$ IP(t_j), Min(TOKEN$_{\text{arr}}(p_j)$) and Max(TOKEN$_{\text{arr}}(p_j)$) can be used to represent the lower and upper bound of TOKEN$_{\text{arr}}(p_j)$. According to formulas (3.1) and (3.2), we can define EFBT'(t_j), LFET'(t_j), EFBT"(t_j) and LFET"(t_j) as follows:

$$\text{EFBT'}(t_j) = \text{Max}\{\text{Min(TOKEN}_{\text{arr}}(p_j)) + \text{TC}_{\text{min}}(p_j), \text{Max}\{\text{Min(TOKEN}_{\text{arr}}(p_j)) : p_j \in \text{IP}(t_j)\} + \text{TC}_{\text{min}}(t_j)\} \tag{3.3}$$

$$\text{LFET'}(t_j) = \text{Min}\{\text{Max(TOKEN}_{\text{arr}}(p_j)) + \text{TC}_{\text{max}}(p_j), \text{Max}\{\text{Max(TOKEN}_{\text{arr}}(p_j)) : p_j \in \text{IP}(t_j)\} + \text{TC}_{\text{max}}(t_j)\} \tag{3.4}$$

$$\text{EFBT"}(t_j) = \text{Max}\{\text{Max(TOKEN}_{\text{arr}}(p_j)) + \text{TC}_{\text{min}}(p_j), \text{Max}\{\text{Max(TOKEN}_{\text{arr}}(p_j)) : p_j \in \text{IP}(t_j)\} + \text{TC}_{\text{min}}(t_j)\} \tag{3.5}$$

$$\text{LFET"}(t_j) = \text{Min}\{\text{Min(TOKEN}_{\text{arr}}(p_j)) + \text{TC}_{\text{max}}(p_j), \text{Max}\{\text{Min(TOKEN}_{\text{arr}}(p_j)) : p_j \in \text{IP}(t_j)\} + \text{TC}_{\text{max}}(t_j)\} \tag{3.6}$$

and

$$\text{EFBT'}(t_j) \le \text{EFBT}(t_j) \le \text{EFBT"}(t_j);$$
$$\text{LFET"}(t_j) \le \text{LFET}(t_j) \le \text{LFET'}(t_j).$$

The schedulability of individual transition can be checked as following three cases:
Case 1: For a transition t_j with one input place p_j:

$$\begin{aligned}\text{EFBT}(t_j) =&\text{TOKEN}_{\text{arr}}(p_j)+\\ &\text{Max}\{\text{TC}_{\text{min}}(p_j), \text{TC}_{\text{min}}(t_j)\};\\ \text{LFET}(t_j) =&\text{TOKEN}_{\text{arr}}(p_j)+\\ &\text{Min}\{\text{TC}_{\text{max}}(p_j), \text{TC}_{\text{max}}(t_j)\};\end{aligned}$$

So the result of (LFET(t_j) − EFBT(t_j)) is not influenced by the value of TOKEN$_{\text{arr}}(p_j)$.
Case 2: For a transition t_j with several input places p_j $(j = 1, \cdots, k)$:
Based on Definition 4, we can get LFET"(t_j) − EFBT"(t_j) $\le$ LFET(t_j) − EFBT(t_j) $\le$ LFET'(t_j) − EFBT'(t_j).

- If LFET"(t_j) − EFBT"(t_j) $\ge$ FIRE$_{\text{dur}}(t_j)$ $\Rightarrow$ LFET(t_j) − EFBT(t_j) $\ge$ FIRE$_{\text{dur}}(t_j)$, then t_j is strongly schedulable.
- If LFET"(t_j) − EFBT"(t_j) $<$ FIRE$_{\text{dur}}(t_j)$ and LFET'(t_j)−EFBT'(t_j) $\ge$ FIRE$_{\text{dur}}(t_j)$, then t_j is perhaps strongly schedulable or not. In this case, t_j is in an unsafe state.
- If LFET'(t_j) − EFBT'(t_j) $<$ FIRE$_{\text{dur}}(t_j)$ $\Rightarrow$ LFET(t_j) − EFBT(t_j) $<$ FIRE$_{\text{dur}}(t_j)$, then t_j can not be strongly schedulable.

Case 1 and Case 2 are the sufficient conditions to determine the schedulability of individual transition.
Case 3: For a transition t_j $(j = 1, 2, \cdots, k)$ in a conflict structure:

The strength of TCPNs over other time-related Petri nets is in the modeling and analysis of conflict structures, the handling of any transition in a conflict structure is same as that of a transition with one input place.

3.3 Schedulability Analysis of a Transition Sequence

In TCPN, a marking M_n is said to be reachable if there exists a firing sequence $\sigma = (M_0 t_1 M_1 \cdots t_i M_i \cdots t_n M_n)$, or simply transition sequence $\delta = (t_1 \cdots t_i \cdots t_n)$ that transforms M_0 to M_n, and δ is schedulable with respect to imposed time constraints.

Definition 5. A transition sequence $\delta = (t_1 \cdots t_i \cdots t_n)$ is schedulable if and only if all transitions including in δ are strongly schedulable.

In schedulability analysis of a transition sequence,there exists a Root_Time for each transition t_i, it means that the token arrival times of input places of t_i can be represented by Root_Time uniformly. The schedulability of $\delta = (t_1 \cdots t_n)$ will be checked as follows:

Step1: Determine the initial marking M_0 and suppose TOKEN$_{\text{arr}}(M_0)$ is T_0.

Step2: According to the sequence δ, find the input places of t_i $(i = 1, \cdots, n)$, identify the Root_Time of t_i and token arrival time of input places, and then check the schedulability of t_i. If t_i is strongly schedulable, then do Step 3; else, the unreasonable timing constraints should be modified to make t_i re-schedulable.

Step3: FIRE$_{\text{end}}(t_i)$ should be computed for t_i.

- For transition t_i with one input place:

$$\text{FIRE}_{\text{end}}(t_i) \in [\text{EFBT}(t_i) + \text{FIRE}_{\text{dur}}(t_i), \text{LFET}(t_i)];$$

- For transition t_i with several input places:

$$\text{FIRE}_{\text{end}}(t_i) \in [\text{EFBT'}(t_i) + \text{FIRE}_{\text{dur}}(t_i), \text{LFET'}(t_i)].$$

In Fig. 3, the schedulability of $\delta = (t_1 t_2 t_3 t_4 t_5)$ is checked as follows: Obviously, t_1, t_2, t_3 and t_4 are strongly schedulable. the Root_Time of t_5 is T_2, which represents the firing end time of t_2, token arrive times of p_5 and p_6 are $[T_2 + 6, T_2 + 7]$ and $[T_2 + 8, T_2 + 9]$ respectively. Based on schedulability of transition with several input places, t_5 is also strongly schedulable, thus δ is schedulable based on imposed time, the marking $M_n = \{p_9\}$ is reachable in TCPN. Suppose T_0 be the beginning time of total process, FIRE$_{\text{end}}(t_i)$ based on T_0 can be computed, the ending time

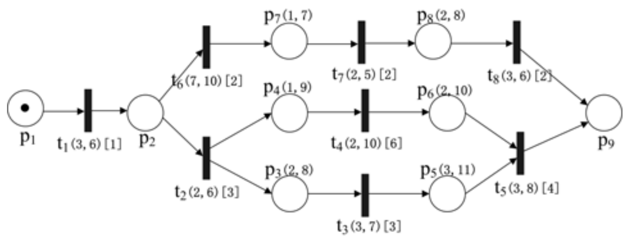


Figure 3. Schedulability of a transition sequence

of task sequence δ:TOKEN$_{\text{arr}}(p_9)$ is $[T_0 + 24, T_0 + 29]$, which means that if δ could complete its execution successfully, it can't end before $T_0 + 24$ or after $T_0 + 29$.

In a workflow modeling with TCPN, transition sequence δ can represent a task sequence. If δ is schedulable, for every task or transition $t_i \in \delta$, LFET$(t_i) -$ EFBT$(t_i) \geq$ FIRE$_{\text{dur}}(t_i)$, there must exist a decision span for each task at run-time, thus the total task sequence δ can complete the execution successfully.

4 Compositional Analysis of Schedulability

In this section, we describe how to conduct schedulability analysis by decomposing a firing sequence in UN into a number of subsequences. We use EN(M) to denote the set of transitions enabled under marking M. According to [2], we can get the Definition 6:

Definition 6. Let $\sigma_1 = (M_{10}t_{11}M_{11}\cdots t_{1i}M_{1i}\cdots t_{1m}$ $M_{1m})$ $(m \geq 1)$ and $\sigma_2 = (M_{20}t_{21}M_{21}\cdots t_{2j}M_{2j}\cdots t_{2n}$ $M_{2n})$ $(n \geq 1)$ be two sequences in UN, where M_{10} and M_{20} are reachable from M_0. σ_2 is composable with σ_1 if and only if $M_{1m} = M_{20}$ and EN$(M_{1m}) \cap$ EN$(M_{1m-1}) - \{t_{1m}\} = \varnothing$. The composition of σ_2 with σ_1, denoted as $\sigma_1 + \sigma 2$, is

$$(M_{10}t_{11}M_{11}\cdots t_{1i}M_{1i}\cdots t_{1m}M_{1m}t_{21}M_{21} \\ \cdots t_{2j}M_{2j}\cdots t_{2n}M_{2n})$$

Obviously, according to Definition 6, it is incorrect to separate concurrent transitions while decomposing a sequence.

Theorem 3. Let σ_2 be composable with σ_1. $\delta_1\delta_2$ is schedulable if and only if both δ_1 and δ_2 are schedulable.

Proof. Let
$$\begin{aligned}
\sigma_1 &= (M_{10}t_{11}M_{11}\cdots t_{1i}M_{1i}\cdots t_{1m}M_{1m}) \\
\sigma_2 &= (M_{20}t_{21}M_{21}\cdots t_{2j}M_{2j}\cdots t_{2n}M_{2n}) \\
\delta_1 &= (t_{11}\cdots t_{1i}\cdots t_{1m}) \\
\delta_2 &= (t_{21}\cdots t_{2j}\cdots t_{2n}) \\
\delta_1\delta_2 &= (t_{11}\cdots t_{1i}\cdots t_{1m}t_{21}\cdots t_{2j}\cdots t_{2n}) \\
\text{AD}_i &= [\text{EFBT}(t_i), \text{LFET}(t_i)]
\end{aligned}$$

1. Suppose both δ_1 and δ_2 are schedulable. So there exist two sequences of absolute firing domains for checking the schedulability of δ_1 and δ_2, say, $(\text{AD}_{11}\cdots\text{AD}_{1i}\cdots\text{AD}_{1m})$, $(\text{AD}_{21}\cdots\text{AD}_{2j}\cdots\text{AD}_{2n})$, and for each AD$_i$, there is: LFET$(t_i) -$ EFBT$(t_i) \geq$ FIRE$_{\text{dur}}(t_i)$, Since σ_2 is composable with σ_1, so $M_{1m} = M_{20}$, $(\text{AD}_{11}\cdots\text{AD}_{1i}\cdots\text{AD}_{1m}\text{AD}_{21}\cdots\text{AD}_{2j}\cdots\text{AD}_{2n})$ is exactly the sequence of absolute firing domains for checking the schedulability of $\delta_1\delta_2$. Thus $\delta_1\delta_2$ is schedulable.

2. Suppose $\delta_1\delta_2$ is schedulable. There exists a sequence of absolute firing domains for checking the schedulability of $\delta_1\delta_2$, say, $(\text{AD}_{11}\cdots\text{AD}_{1i}\cdots\text{AD}_{1m}\text{AD}_{21}\cdots\text{AD}_{2j}\cdots\text{AD}_{2n})$. Obviously, $(\text{AD}_{11}\cdots\text{AD}_{1i}\cdots\text{AD}_{1m})$ and $(\text{AD}_{21}\cdots\text{AD}_{2j}\cdots\text{AD}_{2n})$ are sequences of absolute firing domains for checking δ_1 and δ_2, respectively. So, both δ_1 and δ_2 are schedulable.

□

Theorem 4. Let σ_i $(1 \leq i \leq k)$ be sequences, and σ_i $(2 \leq i \leq k)$ be composable with σ_{i-1}. $\delta_1\cdots\delta_k$ is schedulable if and only if δ_i $(1 \leq i \leq k)$ are all schedulable.

Proof. It is obvious if $k = 2$; if $k = 3$, then $\delta_1\delta_2\delta_3 = (\delta_1\delta_2)\delta_3$. Let $\delta = \delta_1\delta_2$, $\delta_1\delta_2\delta_3 = \delta\delta_3$. $\delta\delta_3$ is schedulable iff δ and δ_3 are schedulable iff δ_1, δ_2, and δ_3 are schedulable. Suppose $\delta = \delta_1\cdots\delta_{k-1}$ is schedulable iff δ_i $(1 \leq i \leq k-1)$ are all schedulable. $\delta_1\cdots\delta_k = \delta\delta_k$. $\delta_1\cdots\delta_k$ is schedulable iff δ and δ_k are schedulable iff δ_i $(1 \leq i \leq k)$ are all schedulable. □

Theorem 5. Suppose sequence σ_2 is composable with itself (called self-composable) and with sequence σ_1 and sequence σ_3 is composable with σ_2. Let $\delta = (\delta_2)^k = \delta_2\cdots\delta_2\cdots\delta_2$, where the number of δ_2 is k $(k > 0)$. $\delta_1\delta\delta_3$ is schedulable if and only if $\delta_1\delta_2\delta_3$ is schedulable.

Proof. If $\sigma_2 = (M_{20}t_{21}M_{21}\cdots t_{2i}M_{2i}\cdots t_{2m}M_{2m})$ is a self-composable sequence, then $M_{20} = M_{2m}$. For each t_i in δ_2, the schedulability of t_i does not change with times of execution of δ_2 increasing, thus the schedulability of $(\delta_2)^k$ is same as the schedulability of δ_2, and according to Theorem 5, we can easily conclude this theorem. □

Theorem 4 and Theorem 5 can simplify the schedulability analysis of sequences containing loops. Note that Theorems 3-5 are useful only if sequences can be decomposed, i.e., no concurrency is present at decomposition markings.

5 Schedulabililty Analysis of a Workflow Example Using TCPN-PIPE2

5.1 Description of TCPN-PIPE2

This subsection gives a brief description of TCPN-PIPE2, which is developed based on an open-source software: Platform Independent Petri Net Editor2(PIPE2). We have applied the ideas of compositional schedulability analysis to TCPN-PIPE2, the intent of which is to build, simulate, and analyze TCPN models.

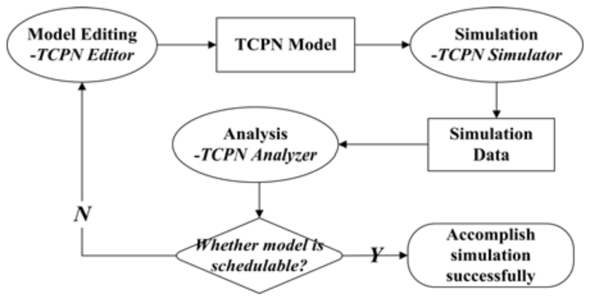


Figure 4. Phases of schedulability analysis process

Fig. 4 schematizes the schedulability analysis process implemented through TCPN-PIPE2. Each ellipse represents a phase of the process and reports the name of each module that supports it, we now give a more detail description of the modules contained in TCPN-PIPE2.

TCPN Editor: Supports direct model construction in the TCPN formalism. The components in a TCPN model can be directly modeled in this graphical user interface, and automatically generates TCPN models encoded in XML format.

TCPN Simulator: Supports the simulation of TCPN models by moving the tokens on the firing of transitions, and performs simulation in step-by-step mode. The result of simulation is a list of simulation data such as: schedulability of transitions, Fire_End of transitions.

TCPN Analyzer: It has the duty of analyzing the simulation data, and determining the schedulability of specified transition sequence. If sequence is not schedulable, the Simulator can't proceed until the time constraints are modified.

5.2 Comparisons

In [2], xu *et al.*proposed the compositional technique, but this technique is based upon a totally different model: TPNs. Compared with TPNs, TCPNs have distinctive time constraints and transition firing rules; Moreover, the schedulability analysis of TCPNs is different from that of TPNs.

There are many other famous time-related tools, such as TimeNET and F-net for stochastic Petri nets(SPNs), ORIS, TINA and Opera for TPNs and so on. Here we take TimeNet [10] and ORIS [11] for example, ORIS is a tool for analyzing extended TPN models, named Preemptive TPNs; TimeNET provides a unified framework for modeling and performance evaluation of non-Markovian stochastic Petri nets, they are both based upon different time-related Petri nets models from TCPNs.

Compared with tools listed or unlisted above, the functionality of TCPN-PIPE2 is somewhat limited. i.e., TCPN-PIPE2 can't generate reachability graph, and can only implement the state space analysis of basic Petri nets.

Although the functionality is limited, the expressive semantics of TCPNs gives TCPN-PIPE2 a expressive power, which permits the treatment of practical schedulability problems. This tool provides an experiment platform for future research about TCPNs, we are currently extending the capabilities of TCPN-PIPE2 to analyze TCPN models with more complicated behaviors, even the SPNs and TPNs models.

5.3 Schedulability Analysis Using TCPN-PIPE2

Through TCPN-PIPE2, we modeled various time-dependent workflow architectures. Here, we should consider a famous workflow example: the processing of complaints [6].

The TCPN model of this workflow is built in TCPN Editor and shown in Fig. 5, in TCPN-PIPE2, we use interval $(0, 1000)$ to represent $(0, \infty)$. Without consideration of timing constraints, the functional requirements of above cases are easily analyzed. For example, $\delta = (t_1\gamma_4 t_8 t_9 \gamma^k t_{11} t_{12})$ $\gamma_4 = (t_2 t_3 t_5 t_7)$ or $(t_2 t_5 t_3 t_7)$ or $(t_2 t_3 t_7 t_5)$ or $(t_3 t_2 t_5 t_7)$ or $(t_3 t_2 t_7 t_5)$ or $(t_3 t_7 t_2 t_5)$, $\gamma = (t_{10} t_8 t_9)$;
is one of the four basic transition sequences in UN that model the functional behaviors of correspondent workflow. These four sequences can transform M_0 to $M_n = \{p_{11}\}$.

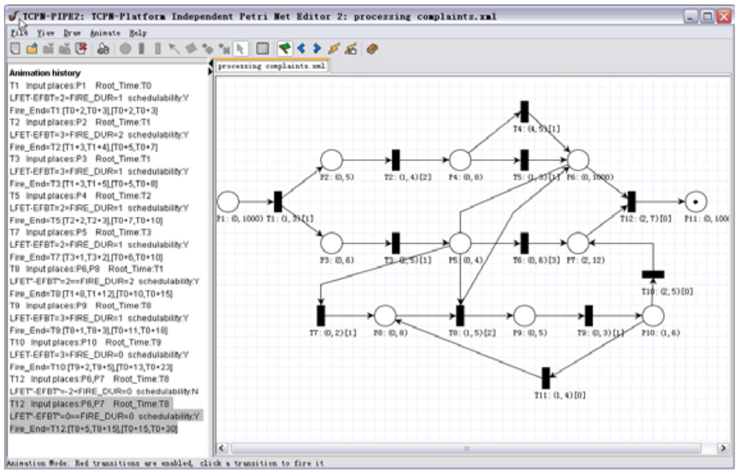
Next, we will present the schedulability analysis of δ, according to Theorem 5, because γ is self-composable, to check the schedulability of δ, we only need to check the schedulability of $\delta_0 = (t_1\gamma_4 t_8 t_9 t_{11} t_{12})$ $(k = 0)$ and $\delta_1 = (t_1\gamma_4 t_8 t_9 \gamma t_{11} t_{12})$ $(k = 1)$, suppose $\gamma_4 = (t_2 t_3 t_5 t_7)$.

After modifying timing constraints of t_8 and t_{12} from $(1, 5)$[2],$(2, 5)$[0] to $(1, 8)$[2],$(2, 7)$[0], δ_0 and δ_1 are both schedulable, then δ is schedulable.

Table 1. FIRE_END of transitions in δ

Transition	Condition	FIRE_END
t_1	null	$[T_0 + 2, T_0 + 3]$
t_2	null	$[T_0 + 5, T_0 + 7]$
t_3	null	$[T_0 + 5, T_0 + 8]$
t_5	null	$[T_0 + 7, T_0 + 10]$
t_7	null	$[T_0 + 6, T_0 + 10]$
t_8	$k \geq 0$	$[T_0 + 10 + 5 \times k, T_0 + 18 + 15 \times k]$
t_9	$k \geq 0$	$[T_0 + 11 + 5 \times k, T_0 + 21 + 15 \times k]$
t_{10}	$k \geq 1$	$[T_0 + 12 + 5 \times (k-1), T_0 + 25 + 15 \times (k-1)]$
t_{11}	$k \geq 0$	$[T_0 + 13 + 5 \times k, T_0 + 26 + 15 \times k]$
t_{12}	$k \geq 0$	$[T_0 + 10 + 5 \times (k+1), T_0 + 18 + 15 \times (k+1)]$

Table. 1 shows the Fire_End of transitions in δ based on modified time constraints, k is the times that γ executes, the value shown in table 1 is validated by TCPN-PIPE2. Similarly, other three sequences are also schedulable with respect to newly-modified timing constraints, the



```
Animation history
T1   Input places:P1    Root_Time:T0
LFET-EFBT=2>FIRE_DUR=1  schedulability:Y
Fire_End=T1:[T0+2,T0+3],[T0+2,T0+3]
T2   Input places:P2    Root_Time:T1
LFET-EFBT=3>FIRE_DUR=2  schedulability:Y
Fire_End=T2:[T1+3,T1+4],[T0+5,T0+7]
T3   Input places:P3    Root_Time:T1
LFET-EFBT=3>FIRE_DUR=1  schedulability:Y
Fire_End=T3:[T1+3,T1+5],[T0+5,T0+8]
T5   Input places:P4    Root_Time:T2
LFET-EFBT=2>FIRE_DUR=1  schedulability:Y
Fire_End=T5:[T2+2,T2+3],[T0+7,T0+10]
T7   Input places:P5    Root_Time:T3
LFET-EFBT=2>FIRE_DUR=1  schedulability:Y
Fire_End=T7:[T3+1,T3+2],[T0+6,T0+10]
T8   Input places:P6,P8    Root_Time:T1
LFET*-EFBT*=2==FIRE_DUR=2  schedulability:Y
Fire_End=T8:[T1+8,T1+12],[T0+10,T0+15]
T9   Input places:P9    Root_Time:T8
LFET-EFBT=3>FIRE_DUR=1  schedulability:Y
Fire_End=T9:[T8+1,T8+3],[T0+11,T0+18]
T10   Input places:P10    Root_Time:T9
LFET-EFBT=3>FIRE_DUR=0  schedulability:Y
Fire_End=T10:[T9+2,T9+5],[T0+13,T0+23]
T12   Input places:P6,P7    Root_Time:T8
LFET*-EFBT*=-2<FIRE_DUR=0  schedulability:N
T12   Input places:P6,P7    Root_Time:T8
LFET*-EFBT*=0==FIRE_DUR=0  schedulability:Y
Fire_End=T12:[T8+5,T8+15],[T0+15,T0+30]
```

```
Animation Mode: Red transitions are enabled, click a transition to fire it
```

Figure 5. TCPN model of the workflow

final marking $M_n = \{p_{11}\}$ is reachable.

6 Conclusions and Future Work

The contributions of this paper include:

1. A detailed approach to schedulability analysis of individual transition and transition sequences.
2. A compositional technique to analyze the schedulability of sequences with loops.
3. An analyzing tool for TCPN models: TCPN-PIPE2

An interesting research problem is how the compositional strategy can be extended to analyze nets with irregular structures. Since there exist some weaknesses in TCPN-PIPE2, extending the capabilities of this tool is our on-going research task.

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